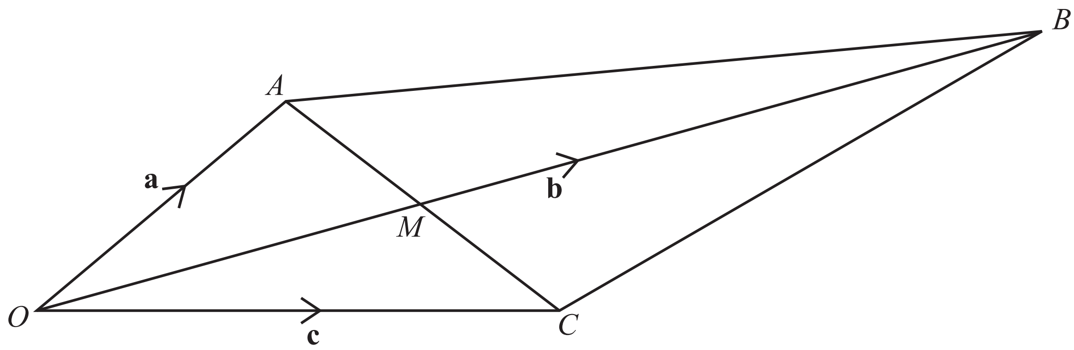
1 **(a)**



The diagram shows a figure $OABC$, where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The lines AC and OB intersect at the point M where M is the midpoint of the line AC.

(i) Find, in terms of **a** and **c**, the vector $\overrightarrow{OM}$. [2]

(ii) Given that $OM : MB = 2:3$, find **b** in terms of **a** and **c**. [2]

(b) Vectors **i** and **j** are unit vectors parallel to the x-axis and y-axis respectively.

The vector **p** has a magnitude of 39 units and has the same direction as $-10\mathbf{i}+24\mathbf{j}$.

(i) Find **p** in terms of **i** and **j**. [2]

(ii) Find the vector **q** such that $2\mathbf{p}+\mathbf{q}$ is parallel to the positive y-axis and has a magnitude of 12 units. [3]

(iii) Hence show that $|\mathbf{q}| = k\sqrt{5}$, where k is an integer to be found. [2]

2 **(a)** Given that $\mathbf{p} = 2\mathbf{i} - 5\mathbf{j}$ and $\mathbf{q} = \mathbf{i} - 3\mathbf{j}$, find the unit vector in the direction of $3\mathbf{p} - 4\mathbf{q}$. [4]

3 Vectors **a**, **b** and **c** are such that $\mathbf{a} = \begin{pmatrix} 2 \\ y \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$.

(i) Given that $|\mathbf{a}| = |\mathbf{b} - \mathbf{c}|$, find the possible values of y. [3]

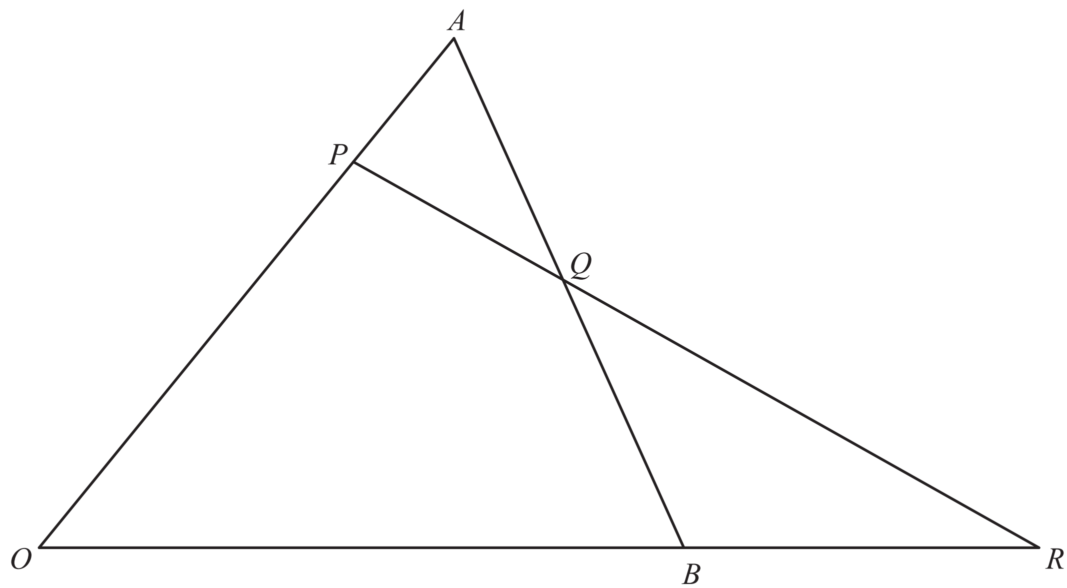
(ii) Given that $\mu(\mathbf{b} + \mathbf{c}) + 4(\mathbf{b} - \mathbf{c}) = \lambda(2\mathbf{b} - \mathbf{c})$, find the value of μ and of λ. [3]

4 Vectors **i** and **j** are unit vectors parallel to the x-axis and y-axis respectively.

(a) The vector **v** has a magnitude of $3\sqrt{5}$ units and has the same direction as $\mathbf{i} - 2\mathbf{j}$. Find **v** giving your answer in the form $a\mathbf{i} + b\mathbf{j}$, where a and b are integers. [2]

(b) The velocity vector **w** makes an angle of 30° with the positive x-axis and is such that $|\mathbf{w}| = 2$. Find **w** giving your answer in the form $\sqrt{c}\,\mathbf{i} + d\mathbf{j}$, where c and d are integers. [2]

5



The diagram shows a triangle OAB such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point P lies on OA such that $OP = \frac{3}{4}OA$. The point Q is the mid-point of AB. The lines OB and PQ are extended to meet at the point R. Find, in terms of $\mathbf{a}$ and $\mathbf{b}$,

(a) $\overrightarrow{AB}$, [1]

(b) $\overrightarrow{PQ}$. Give your answer in its simplest form. [3]

It is given that $n\overrightarrow{PQ} = \overrightarrow{QR}$ and $\overrightarrow{BR} = k\mathbf{b}$, where n and k are positive constants.

(c) Find $\overrightarrow{QR}$ in terms of n, **a** and **b**. [1]

(d) Find $\overrightarrow{QR}$ in terms of k, **a** and **b**. [2]

(e) Hence find the value of n and of k. [3]

6 **(a)** The vector **v** has a magnitude of 39 units and is in the same direction as $\begin{pmatrix} -12 \\ 5 \end{pmatrix}$. Write **v** in the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where a and b are constants. [2]

(b) Vectors **p** and **q** are such that $\mathbf{p} = \begin{pmatrix} r+s \\ r+6 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 5r+1 \\ 2s-1 \end{pmatrix}$, where r and s are constants. Given that $2\mathbf{p} + 3\mathbf{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, find the value of r and of s. [4]

7 **(a)** Find the unit vector in the direction of $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$. [1]

(b) Given that $\begin{pmatrix} 4 \\ 1 \end{pmatrix} + k\begin{pmatrix} -2 \\ 3 \end{pmatrix} = r\begin{pmatrix} -10 \\ 5 \end{pmatrix}$, find the value of each of the constants k and r. [3]

(c) Relative to an origin O, the points A, B and C have position vectors $\mathbf{p}$, $3\mathbf{q}-\mathbf{p}$ and $9\mathbf{q}-5\mathbf{p}$ respectively.

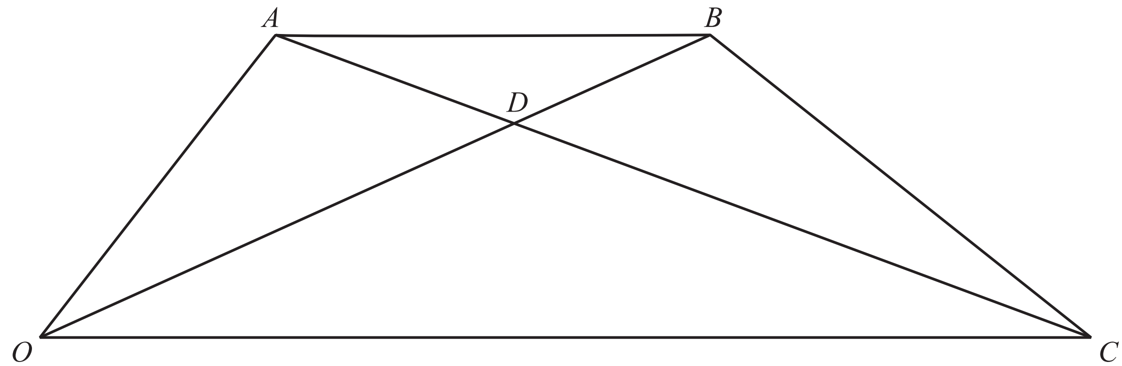
(i) Find $\overrightarrow{AB}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. [1]

(ii) Find $\overrightarrow{AC}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. [1]

(iii) Explain why A, B and C all lie in a straight line. [1]

(iv) Find the ratio $AB : BC$. [1]

8



The diagram shows a quadrilateral $OABC$. The point D lies on OB such that $\overrightarrow{OD} = 2\overrightarrow{DB}$ and $\overrightarrow{AD} = m\overrightarrow{AC}$, where m is a scalar quantity.

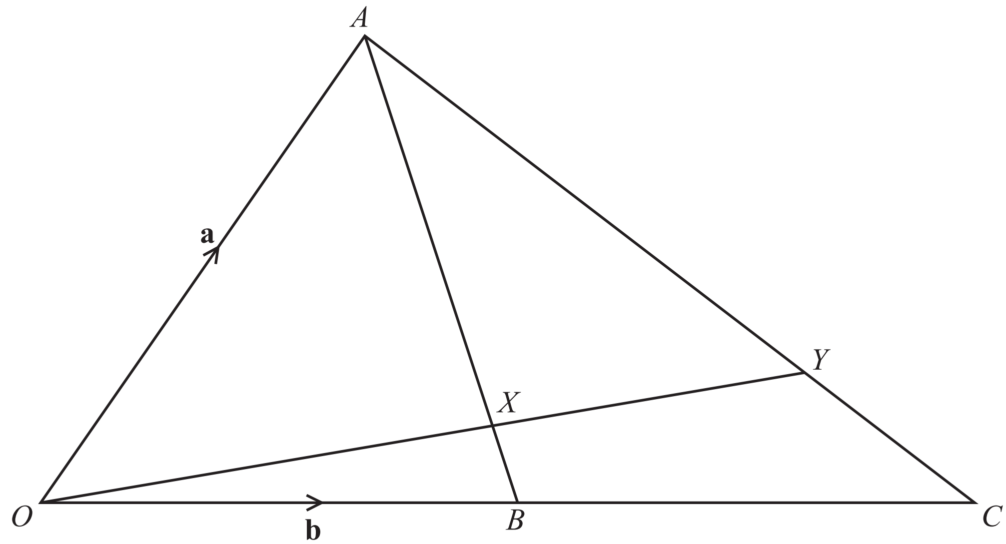
$$\overrightarrow{OA} = \mathbf{a} \qquad \overrightarrow{OB} = \mathbf{b} \qquad \overrightarrow{OC} = \mathbf{c}$$

(i) Find $\overrightarrow{AD}$ in terms of m, $\mathbf{a}$ and $\mathbf{c}$. [1]

(ii) Find $\overrightarrow{AD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(iii) Given that $15\mathbf{a} = 16\mathbf{b} - 9\mathbf{c}$, find the value of m. [3]

9



The diagram shows the triangle OAC. The point B is the midpoint of OC. The point Y lies on AC such that OY intersects AB at the point X where $AX:XB = 3:1$. It is given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Find $\overrightarrow{OX}$ in terms of **a** and **b**, giving your answer in its simplest form. [3]

(b) Find $\overrightarrow{AC}$ in terms of **a** and **b**. [1]

(c) Given that $\overrightarrow{OY} = h\overrightarrow{OX}$, find $\overrightarrow{AY}$ in terms of **a**, **b** and h. [1]

(d) Given that $\overrightarrow{AY} = m\overrightarrow{AC}$, find the value of h and of m. [4]

Question 1

1 (a)(i)	$\overrightarrow{OM} = \overrightarrow{OC} + \frac{1}{2}\left(\overrightarrow{OA} - \overrightarrow{OC}\right)$ oe	**M1**	may be implied by correct answer.
	$\frac{1}{2}(\mathbf{a}+\mathbf{c})$	**A1**	

(a)(ii)	$\mathbf{b} = \frac{5}{2}\overrightarrow{OM}$ oe , $\frac{5}{2}$(*their* (i)) or $\overrightarrow{OM} = \frac{2}{3}\left(\mathbf{b} - \overrightarrow{OM}\right)$	**M1**	dealing with ratio correctly to relate **b** or $\overrightarrow{OB}$ to $\overrightarrow{OM}$
	$= \frac{5}{4}(\mathbf{a}+\mathbf{c})$	**A1**	
(b)(i)	$\lvert -10\mathbf{i} + 24\mathbf{j} \rvert = 26$ $\mathbf{p} = \frac{39}{26}(-10\mathbf{i} + 24\mathbf{j})$	**M1**	magnitude of $-10\mathbf{i} + 24\mathbf{j}$ and use with 39
	$\mathbf{p} = -15\mathbf{i} + 36\mathbf{j}$	**A1**	
(b)(ii)	If parallel to the y-axis, **i** component is zero	**M1**	realising **i** component is zero
	so $2\mathbf{p} + \mathbf{q} = 12\mathbf{j}$	**DM1**	use of 12
	$\mathbf{q} = 30\mathbf{i} - 60\mathbf{j}$	**A1**	
(b)(iii)	$\lvert \mathbf{q} \rvert = 30\sqrt{1^2 + (-2)^2}$ or $\sqrt{900} \times \sqrt{5}$	**M1**	attempt at magnitude of *their* **q**
	$\lvert \mathbf{q} \rvert = 30\sqrt{5}$	**A1**	Answer Given: must have full and correct working

Question 2

2 (a)	$3(2\mathbf{i} - 5\mathbf{j}) - 4(\mathbf{i} - 3\mathbf{j})$	**M1**	For expansion and collection of terms
	$3\mathbf{p} - 4\mathbf{q} = 2\mathbf{i} - 3\mathbf{j}$	**A1**	
	Magnitude of their $2\mathbf{i} - 3\mathbf{j}$ $\sqrt{2^2 + (-3)^2}$	**M1**	For method to find magnitude
	Unit vector $= \dfrac{2\mathbf{i} - 3\mathbf{j}}{\sqrt{13}}$	**A1**	

Question 3

3 (i)	$\mathbf{b}-\mathbf{c}=\begin{pmatrix}6\\-2\end{pmatrix}$	B1	may be implied by further correct working
		M1	for one correct attempt at using the modulus
	$4+y^2=36+4$ $y=\pm 6$	A1	
(ii)	$\mu+4=2\lambda$ or $-4\mu+24=7\lambda$ $\mu-4=-\lambda$ or $8\mu-8=\lambda$	B1 B1	for one correct equation in μ and λ for a second correct equation in μ and λ
	leading to $\mu=\frac{4}{3}$, $\lambda=\frac{8}{3}$ oe allow 1.33 and 2.67 or better	DB1	for both, must have both previous **B** marks

Question 4

Question	Answer	Marks	Partial Marks
4 (a)	$\mathbf{v} = 3\sqrt{5} \times \frac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$	**M1**	attempt to find the magnitude of $(\mathbf{i} - 2\mathbf{j})$ and use
	$= 3\mathbf{i} - 6\mathbf{j}$	**A1**	for $3\mathbf{i} - 6\mathbf{j}$ only
(b)	$\mathbf{w} = 2\cos 30^{\circ}\,\mathbf{i} + 2\sin 30^{\circ}\,\mathbf{j}$	**M1**	attempt to use trigonometry correctly to obtain components
	$= \sqrt{3}\mathbf{i} + \mathbf{j}$	**A1**	

Question 5

5 (a)	$\mathbf{b}-\mathbf{a}$	**B1**	
(b)	$\frac{1}{4}\mathbf{a}+\frac{1}{2}(\mathbf{b}-\mathbf{a})$ or $-\frac{3}{4}\mathbf{a}+\frac{1}{2}(\mathbf{a}+\mathbf{b})$	**B1**	For $\frac{1}{4}\mathbf{a}$ or $-\frac{3}{4}\mathbf{a}$
		B1	For $\frac{1}{2}(\mathbf{b}-\mathbf{a})$ or $\frac{1}{2}(\mathbf{a}+\mathbf{b})$
	$\frac{1}{2}\mathbf{b}-\frac{1}{4}\mathbf{a}$	**B1**	Correct and simplified
(c)	$n\left(\frac{1}{2}\mathbf{b}-\frac{1}{4}\mathbf{a}\right)$	**B1**	**FT** on *their* answer to (b)
(d)	$\frac{1}{2}(\mathbf{b}-\mathbf{a})+k\mathbf{b}$	**M1**	For use of *their* (a) and $k\mathbf{b}$
		A1	

(e)	$\frac{1}{2}(\mathbf{b}-\mathbf{a})+k\mathbf{b}=n\left(\frac{1}{2}\mathbf{b}-\frac{1}{4}\mathbf{a}\right)$ $-\frac{1}{2}=-\frac{n}{4}$ $\frac{1}{2}+k=\frac{n}{2}$	**M1**	For equating *their* (c) and (d) and then equating like vectors to obtain 2 equations
	$n=2$	**A1**	
	$k=\frac{1}{2}$	**A1**	

Question 6

Question	Answer	Marks	Guidance
6 (a)	$\left\lvert\begin{pmatrix}-12\\5\end{pmatrix}\right\rvert=13$	**B1**	For magnitude, may be implied by a correct **v**
	$\mathbf{v}=\begin{pmatrix}-36\\15\end{pmatrix}$ or $3\begin{pmatrix}-12\\5\end{pmatrix}$	**B1**	Must be a vector
(a) Alternative	If $t\left\lvert\begin{pmatrix}-12\\5\end{pmatrix}\right\rvert=39$, $t=3$	**B1**	For value of t, may be implied by a correct **v**
	$\mathbf{v}=\begin{pmatrix}-36\\15\end{pmatrix}$ or $3\begin{pmatrix}-12\\5\end{pmatrix}$	**B1**	
(b)		**M1**	For equating like vectors at least once
	$17r+2s+3=0$ $2r+6s+9=0$	**M1**	**Dep** For solution of resulting equations to obtain 2 solutions
	$r=0$	**A1**	
	$s=-\dfrac{3}{2}$ oe	**A1**	

Question 7

7 (a)	$\frac{1}{13}\begin{pmatrix}5\\-12\end{pmatrix}$	**B1**	
(b)	$4-2k=-10r$ $1+3k=5r$	**M1**	equating like vectors to obtain 2 equations
	$r=-\frac{7}{10},\ k=-\frac{3}{2}$	**M1**	Dep on previous M mark, for attempt to solve simultaneously
		A1	
(c)(i)	$3\mathbf{q}-2\mathbf{p}$	**B1**	
(c)(ii)	$9\mathbf{q}-6\mathbf{p}$	**B1**	
(c)(iii)	A common point of *A* and the same direction vector	**B1**	
(c)(iv)	1:2	**B1**	

Question 8

Question	Answer	Marks	Guidance
8 (i)	$\overrightarrow{AD} = m(\mathbf{c} - \mathbf{a})$	**B1**	
(ii)	$\overrightarrow{AD} = \overrightarrow{OD} - \mathbf{a}$	**B1**	for $\overrightarrow{OD} = \frac{2}{3}\mathbf{b}$
	$= \frac{2}{3}\mathbf{b} - \mathbf{a}$	**B1**	**FT** *their* $\overrightarrow{OD}$ if $\overrightarrow{OD} = k\mathbf{b}$
(iii)	$m(\mathbf{c} - \mathbf{a}) = \frac{2}{3}\mathbf{b} - \mathbf{a}$	**M1**	equating parts **(i)** and **(ii)**
	$24\mathbf{a}(1 - m) + 24m\mathbf{c} = 16\mathbf{b}$ Comparing with $15\mathbf{a} + 9\mathbf{c} = 16\mathbf{b}$	**M1**	attempt to eliminate or compare like vectors using given condition
	$m = \frac{3}{8}$	**A1**	

Question 9

9 (a)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ or $\overrightarrow{BA} = \mathbf{a} - \mathbf{b}$	**B1**	
	$\overrightarrow{OX} = \mathbf{a} + \frac{3}{4}\overrightarrow{AB}$ or $\overrightarrow{OX} = \mathbf{b} + \frac{1}{4}\overrightarrow{BA}$ $\overrightarrow{OX} = \mathbf{a} + \frac{3}{4}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{OX} = \mathbf{b} + \frac{1}{4}(\mathbf{a} - \mathbf{b})$	**M1**	For correct use of ratio, using *their* $\overrightarrow{AB}$ or $\overrightarrow{BA}$
	$\overrightarrow{OX} = \frac{\mathbf{a}}{4} + \frac{3}{4}\mathbf{b}$	**A1**	
(b)	$\overrightarrow{AC} = 2\mathbf{b} - \mathbf{a}$	**B1**	
(c)	$\overrightarrow{AY} = -\mathbf{a} + h\left(\frac{\mathbf{a}}{4} + \frac{3}{4}\mathbf{b}\right)$	**B1**	**FT** on *their* $\overrightarrow{OX}$
(d)	$-\mathbf{a} + h\left(\frac{\mathbf{a}}{4} + \frac{3}{4}\mathbf{b}\right) = m(2\mathbf{b} - \mathbf{a})$	**M1**	For equating appropriate vectors and attempt to equate like vectors
	$-1 + \frac{h}{4} = -m$	**A1**	**FT** from *their* $\overrightarrow{AY}$ and $\overrightarrow{AC}$
	$\frac{3h}{4} = 2m$	**A1**	**FT** from *their* $\overrightarrow{AY}$ and $\overrightarrow{AC}$
	$h = \frac{8}{5}, \quad m = \frac{3}{5}$	**A1**	For both